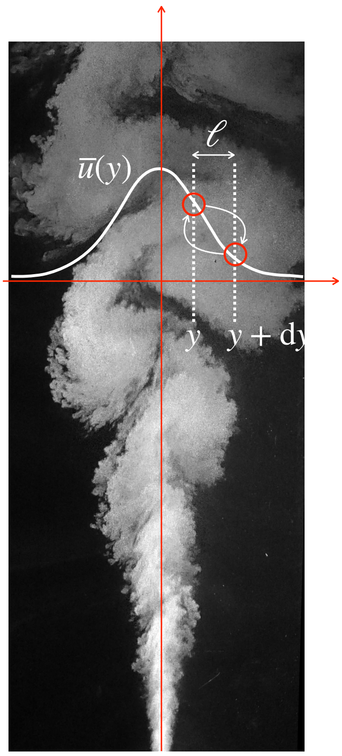
The Taylor-Prandtl controversy: Momentum, Vorticity & Scalar transport in turbulence

E. Villermaux

Aix-Marseille Université
e-mail: emmanuel.villermaux@univ-amu.fr

In 1932, G. I. Taylor wrote a fascinating article suggesting that, in two dimensional turbulent flows at least, it is not the momentum of the eddies which is conserved from one step of their random walk to the other (the so-called Reynolds-Prandtl analogy), but their vorticity, and that therefore the conservation equations for the velocity u and the concentration of a passive scalar c must be *different*. Taylor's 'vorticity transport' theory thus predicts that, across a 2D wake or a jet, the u-profile is exactly related to the c-profile (scaled by their maximal value) by $u(r) = c(r)^2$.

This bold statement invites to reexamine critically the fundamentals of *diffusion* and of *dispersion* in order to distinguish the two, and underline their possible couplings. The discussion will be illustrated by various examples of flows in porous media, dense suspensions, turbulence...



A cut through a plane (2D) turbulent smoke jet.

The outline of the lectures will be†:

1. Historical perspective on the notion of diffusion: Fourier, Fick, Maxwell, Einstein, Langevin, Richardson.
2. The special case of turbulence: Boussinesq, Reynolds, Prandtl, Taylor, or the delicate choice of the 'turbulent viscosity'.
3. Notable examples of couplings between diffusion and dispersion in various media. Batchelor and coarsening scales.
4. A worked-out example: turbulent jets in 2D and 3D with very different Schmidt numbers.
5. The status of the 'turbulent Schmidt number' and lessons from a controversy.

†Relevant references are: TAYLOR, G. I. The transport of vorticity and heat through fluids in turbulent motion. *Proc. Roy. Soc. London A* **135** , 685–705 (1932), VILLERMAUX, E. Mixing versus Stirring. *Annu. Rev. Fluid Mech.* **51** , 245–73 (2019).